

Theoretical Investigation of a Temperature Instability in a Modified Penning Discharge (Q-PIG)

F. KLAN

Institut für Plasmaphysik, Garching bei München, Germany

(Z. Naturforsch. **25 a**, 707–715 [1970]; received 12 February 1970)

An instability connected with strong coherent oscillations of the electron temperature as observed in a PIG discharge is investigated theoretically. The energy equation of the electron gas involving the electronic heat flux as well as certain source terms is linearized. A simple dimensional analysis is used in order to avoid more complicated calculations. The stability criterion obtained in this way is evaluated to give a stability boundary in a certain parameter space. It is also found that the temperature perturbation should move with just the $\mathbf{E} \times \mathbf{B}$ drift velocity of the electrons. The experimental observations agree surprisingly well with the theoretical predictions despite of the crude approximations involved.

1. Introduction

In many gas discharges, in particular in the so-called PIG discharges (PIG = Philips Ionization Gauge), several types of instabilities have been observed. These instabilities were usually identified to be perturbations of the plasma density while variations of the electron temperature in space and time were not considered to be important. The low frequency instabilities observed in PIG discharges (see for instance BONNAL et al.¹, CHEN et al.² or THOMASSEN³) are rather well explained by theories of HOH⁴ and SIMON⁵ which are based on the different drift velocities of ions and electrons moving through the neutral gas. However, some years ago the author observed a new type of low frequency instability in a PIG discharge with a somewhat modified electrode arrangement⁶. The dominant feature of this instability was the surprising fact that the electron temperature rather than the density was oscillating. Similar observations were reported recently by ZAKRZEWSKI, BEAUDRY and CLOUTIER (l. c.^{7–9}) at somewhat higher frequencies. To confirm his previous observations the author carried out a number of additional experiments, some of

which were presented at the Bucharest conference 1969¹⁰. The experimental material has recently been published in greater detail¹¹ and is also available as a laboratory report¹².

The modified PIG discharge differs from conventional PIGs essentially in the following points:

1. There is an extremely stable state having a very low electron temperature. For this reason the discharge is called a “Q-PIG” (Q = quiescent).
2. The radial electric field strength is very small despite the high operating voltage.
3. There is a low frequency instability which causes the electron temperature to vary while the density is constant in time.

From the last point it is clear that theories as those of Hoh or Simon may not be able to explain the observed experimental facts, since they assume a constant electron temperature. There are some theoretical papers which allow the electron temperature to vary, but only very few of them are concerned with weakly ionized plasmas. TIMOFEEV¹³, for instance, considered theoretically the case where both electron density and temperature are perturbed; he added to the familiar fluid equations the energy equation of the electron gas including the

¹ J. F. BONNAL, G. BRIFFORD, and C. MANUS, Phys. Rev. Lett. **6**, 665 [1961].

² F. F. CHEN and A. W. COOPER, Phys. Rev. Lett. **9**, 333 [1962].

³ K. I. THOMASSEN, Phys. Fluids **9**, 626 [1966]; Phys. Rev. Lett. **14**, 587 [1965].

⁴ F. C. HOH, Ark. Fys. **24**, 285 [1963]; Phys. Fluids **6**, 1184 [1963].

⁵ A. SIMON, Phys. Fluids **6**, 382 [1963].

⁶ F. KLAN, Proc. 8th Conf. on Phenomena in Ionized Gases, Vienna 1967, p. 190.

⁷ Z. ZAKRZEWSKI, C. BEAUDRY, and G. G. CLOUTIER, Rev. Sci. Instrum. **39**, 1507 [1968].

⁸ C. BEAUDRY, Z. ZAKRZEWSKI, and G. G. CLOUTIER, Lab. de Phys. des Plasmas, Dept. de Phys., Univ. de Montreal, Rapport No. 68-17.

⁹ Z. ZAKRZEWSKI, C. BEAUDRY, and G. G. CLOUTIER, Lab. de Phys. des Plasmas, Dept. de Phys., Univ. de Montreal, Rapport No. 68-18.

¹⁰ F. KLAN, Proc. 9th Conf. on Phenomena in Ionized Gases, Bucharest 1969, p. 247.

¹¹ F. KLAN, Z. Naturforsch. **25 a**, 263 [1970].

¹² F. KLAN, IPP Rep. No. 3/92, April 1969.

¹³ A. V. TIMOFEEV, Sov. Phys.-Techn. Phys. **7**, 959 [1963].



heat flux but without taking into account any source term. Therefore this treatment, although non-adiabatic, is not complete. Moreover, Timofeev's theory predicts instability only if a transverse temperature gradient exists so that the condition

$$\frac{\partial \ln T_e}{\partial x} > 2 \frac{\partial \ln n_e}{\partial x}$$

holds. In the Q-PIG experiments the unperturbed electron temperature is spatially constant and hence Timofeev's condition is not satisfied. Nevertheless a fluctuating contribution to the electron temperature is observed in the Q-PIG which cannot be explained by Timofeev's theory.

In the theoretical approach presented here not only the electronic heat flux, but also appropriate source terms are involved. This energy balance, based on the discharge properties observed, is proposed and discussed from a general point of view in Sect. 2. This discussion is specified to certain important properties of the source terms in Sect. 3. An appropriate form of an energy equation is found in Sect. 4, which is discussed in some detail in order to account for the special conditions in the Q-PIG. Section 5 is concerned with the linearization of the energy equation which leads to a stability criterion. The results of this theoretical approach are discussed and compared with the results of other authors in Sect. 6 of this paper.

2. The Energy Balance of the Cold Electron Gas

It is well known that in many weakly ionized plasmas the electron velocity distribution is non-Maxwellian¹⁴. In some cases two groups of electrons may well be distinguished in that their mean energies differ appreciably from each other^{15, 16}. Frequently one group having high density and a relatively low mean energy is supposed to have really a Maxwellian velocity distribution. The other group having much lower density but high mean energy moves through the "background" electrons without any appreciable interaction. Especially in PIG discharges a group of high energy electrons is always assumed to be present, but their influence on the discharge equilibrium is usually disregarded. This might possibly be correct in a conventional PIG discharge, where the electron temperature

is sufficiently high to allow for ionization of neutral atoms. In the Q-PIG, however, the cold electrons do by no means have enough energy to ionize or even to excite helium atoms. Therefore the high energy electrons play a very important role in this discharge and they do so in particular with respect to the energy balance.

To understand this situation, let us have a look at Fig. 1 which shows schematically the energy balance of the Q-PIG. We distinguish between "volume processes" which are collisions of various kinds

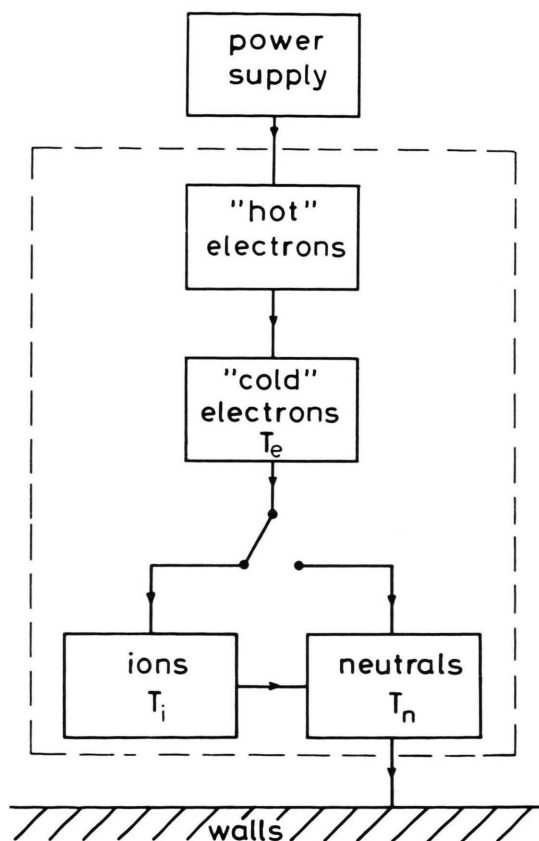


Fig. 1. Schematic representation of the energy balance of the Q-PIG.

occurring inside the plasma volume proper and external processes which affect the plasma from outside. In Fig. 1 the "volume processes" are enclosed by a dotted line. Let us now follow the way the energy takes from the power supply to the ambient walls. The power supply sustains an electric field in front of the cathodes (the cathode fall) in which

¹⁴ W. L. GRANOWSKI, *Der elektrische Strom im Gas*, Akademie-Verlag, Berlin, Vol. I, Chapt. VIII, 1955.

¹⁵ H. SCHLUETER, *Z. Naturforsch.* **16a**, 972 [1961].

¹⁶ E. R. MOSBURG, *Phys. Rev.* **152**, 166 [1966].

electrons emerging from the cathode surface are accelerated. This beam of "hot" electrons intersects the whole plasma volume and heats the "cold" plasma electrons by electron-electron collisions. This process acts as an energy source for the cold electrons and occurs everywhere inside the plasma. The cold electrons themselves give thermal energy to ions and neutrals by elastic collisions. The way the energy divides up between the latter particle species depends on the relative magnitude of the respective collision frequencies. This is indicated in Fig. 1 somewhat oversimplified by a switch. Which position this switch will assume depends very strongly on the electron temperature, whenever the degree of ionization is high enough. In the stable state of the Q-PIG the temperature is so low that for low gas pressure the electron-ion collisions highly predominate and therefore most of the energy is given to the ions. The ions, however, exchange their energy so rapidly with the neutrals that their temperature will be only slightly higher than the temperature of the neutral atoms. Because the neutrals are not confined by the magnetic field, they diffuse freely to the walls carrying the energy out of the plasma volume. This is certainly not the only path of energy through the plasma, but it is the most important one for the energy balance of the cold electron gas; for a more detailed discussion the reader is referred to Ref. ¹².

To confirm the model proposed in Fig. 1 we have to estimate the following two quantities:

1. The rate at which energy is transferred from the hot to the cold electrons, H ;
2. the rate at which energy is transferred from the cold electrons to the ions, C .

For this energy balance to hold, these two quantities must be of the same order of magnitude.

Let us first estimate the order of magnitude of the heating rate in a very simple way. We first ask how many electrons are liberated at the cathode per cm^2 of the surface and per sec. This electron current density, j_e , is related to the current density, j_i , of ions striking the cathode by the coefficient of secondary emission, γ . The cathode current density, j_c , however, is just equal to the sum of j_e and j_i and therefore one obtains easily the electron current density in terms of j_c and γ :

$$j_e = j_c \frac{\gamma}{1+\gamma}. \quad (1)$$

These electrons are now accelerated in the cathode fall, penetrate the plasma volume and lose most of their energy by ionizing or exciting collisions but only a very small amount by collisions with cold electrons. After their energy has dropped below 20 eV, the electrons will no longer ionize or excite. Therefore, the remaining energy has to flow almost completely to the cold electrons, the energy transfer to the heavy particles being small. Thus, with respect to the energy balance of the cold electrons, we start with an electron beam of current density j_e and energy E^* , where E^* is an energy of the order of the excitation energy of the neutral helium atom. Since on average each electron carries this energy into the plasma, the energy flux density is

$$\varepsilon = \frac{j_e}{e} \cdot E^* = \frac{j_c E^*}{e} \cdot \frac{\gamma}{1+\gamma}, \quad (2)$$

where e is the electronic charge. The mean energy flowing into the whole plasma per sec is obtained by multiplying with the effective cathode surface $2F$ (because of the two emitting cathodes of surface F); this divided by the whole plasma volume, FL , yields the mean heating rate, i. e. the power per cm^3 transferred to the cold electrons:

$$H = \frac{2\varepsilon}{L} = \frac{j_c E^*}{eL} \cdot \frac{2\gamma}{1+\gamma} \quad (3)$$

where L is the length of the plasma column. In the Q-PIG, we have $j_c \approx 4 \times 10^{-3} \text{ A/cm}^2$, $L = 50 \text{ cm}$; the coefficient γ is in the case of Molybdenum and helium ions ¹⁷ about 0.25. Taking for E^* the lowest excitation energy of the neutral helium atom ($\approx 20 \text{ eV}$), we obtain:

$$H \approx 0.6 \times 10^4 \text{ erg cm}^{-3} \text{ sec}^{-1}.$$

On the other hand the cooling rate, C , is given by the expression ¹⁸

$$C = 2 \frac{m_e}{m_i} n_e \nu_{ei} \frac{3}{2} k (T_e - T_i), \quad (4)$$

where we have neglected the electron-neutral collisions. For helium ions the mass ratio m_e/m_i is 1.36×10^{-4} . Assuming a density of the order of about 10^{12} cm^{-3} and an electron temperature of 2000°K , ν_{ei} is of the order of 10^8 sec^{-1} . Since the ion temperature should be some 300°K , we obtain as a rough estimate:

$$C \approx 10^4 \text{ erg cm}^{-3} \text{ sec}^{-1}.$$

This is the same order of magnitude as the heating rate H and hence an equilibrium as proposed above could in fact be possible.

¹⁷ H. D. HAGSTRUM, Phys. Rev. **89**, 244 [1953].

¹⁸ S. I. BRAGINSKII, Sov. Phys.-JETP **6**, 358 [1958].

3. Qualitative Discussion of the Energy Balance

We have seen that in the Q-PIG the energy balance of the cold electron gas might be governed by the energy transfer from the hot electrons to the cold ones and from the cold electrons to the ions. Let us now discuss the properties of this equilibrium. Consider first the heating rate H . Because the velocity of the hot electrons is so much greater than that of the cold electrons, the heating rate H will be practically independent of the local temperature of the cold electrons (see also Ref. ¹⁹). This important fact should be kept in mind for the following considerations. On the other hand, this heating rate has to be balanced by a cooling rate, which does depend on the electron temperature. Looking at Eq. (4) and neglecting T_i for the moment, we see that C is proportional to $T_e^{-1/2}$, since ν_{ei} is proportional to $T_e^{-3/2}$. This means that an equilibrium between H and C is possible only for one particular temperature. What happens, however, if by chance this temperature begins to increase? Increasing temperature means decreasing cooling because of the negative exponent of T_e . Since the heating rate is independent of temperature, still the same energy is transferred to the cold electrons, but less energy is flowing away and therefore a net energy input to the cold electrons results. Thus the electron temperature is further increased and hence the cooling rate decreases still more and so on. It is clear that this equilibrium must be unstable. Taking into account finite ion temperature and collisions with neutrals the cooling rate writes:

$$C = 2 \frac{m_e}{m_i} n_e (\nu_{ei} + \nu_{en}) \frac{3}{2} k (T_e - T_i), \quad (5)$$

where we still have assumed that T_i is equal to the neutral gas temperature. In the energy range below 1 eV we may use for ν_{en} an expression proportional ²⁰ to $T_e^{1/2}$. The cooling rate C as given by (5) may easily be evaluated as function of T_e and is shown for typical parameter values (electron density, gas pressure) in Fig. 2 (solid curve). Also shown in this figure is the heating rate (solid horizontal line). Evidently there are now three possible equilibrium temperatures, T_1 , T_2 , and T_3 , but only two of them, T_1 and T_3 , are stable. However, the temperature T_1 is so low that the plasma would rapidly decay due to the very strong volume recombina-

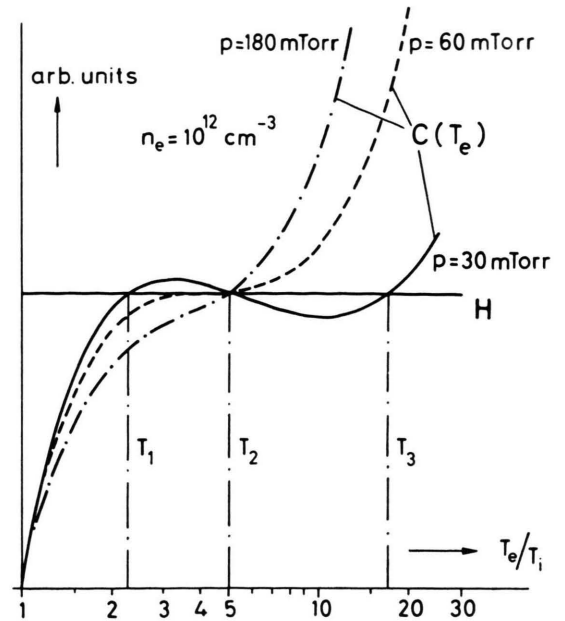


Fig. 2. Energy balance of the cold electron gas at equilibrium as function of T_e (parameter is the neutral gas pressure).

tion. One would therefore expect T_3 to be the actual equilibrium temperature. This is not the case, however, but one finds experimentally a temperature very close to T_2 , which should be highly unstable. The apparent contradiction may be explained in the following way: Consider a cross section of the plasma column as shown in Fig. 3. Assume a region having a temperature somewhat higher ($T + \Delta T$) than in the remaining part of the plasma (T). The temperature gradients produce a heat flux perpendicular to the magnetic field. Because of the unstable nature of the equilibrium between H and C , the temperature within the perturbed region tends

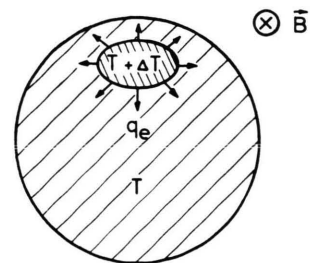


Fig. 3. Cross section of the plasma column having an inhomogeneous, non-symmetric temperature distribution (q_e is the transverse heat flux).

¹⁹ K. D. COLE, Plan. Space Sci. **15**, 873 [1967].

²⁰ S. C. BROWN, Basic Data of Plasma Phys., MIT-Press 1961.

to grow, but the heat flux works against this growth, thus stabilizing the temperature. If the magnetic field is relatively weak, the heat flux is strong enough to suppress the local growth of the electron temperature, whereas at high magnetic field the unstable behaviour of the system at T_2 predominates.

The solid curve shown in Fig. 2 is the cooling rate at a relatively low gas pressure. At higher pressures the collisions between electrons and neutrals become more and more important. In this case the function $C(T_e)$ looks as shown in Fig. 2 for two different pressures (dotted curves). Now only one equilibrium temperature results which is always stable. In this way the gas pressure is found to be another parameter which determines whether the discharge will be stable or unstable with respect to the electron temperature. It is the magnetic field strength and the gas pressure which are varied also in the experiments^{11, 12} in order to drive the discharge from the stable to the unstable state. In the following section we investigate the development of a temperature perturbation including the effect which the magnetic field has on the electronic heat flux.

4. Some Considerations Concerning the Electronic Energy Equation

The experimental observations, which have to be explained, are characterized by fluctuations of the electron temperature. Therefore, the most important relation will be the energy equation of the electron gas. Due to the very low electron temperature but rather high electron density the collisions of the electrons with the ions are much more frequent than those with neutrals. If so, the electrons move almost as if in a fully ionized plasma. Furthermore, as already mentioned, the energy of the cold electrons is so low that only elastic collisions with the helium atoms may occur. These collisions affect not only the transport of momentum, but also that of energy. Therefore, we should use transport coefficients as obtained solving the complete set of kinetic equations. Fortunately this is not necessary here as may be deduced from the relative magnitude of the respective collision frequencies. Instead, we will start from a set of moment equations derived for a fully ionized plasma accounting for the contributions due to electron-neutral collisions by appropriate correction terms.

The energy equation for the electrons of a fully ionized plasma is given, for instance, by BRAGINSKII¹⁸:

$$\frac{3}{2} n \frac{\partial T_e}{\partial t} + \frac{3}{2} n \mathbf{v}_e \cdot \nabla T_e + n T_e \nabla \cdot \mathbf{v}_e + \nabla \cdot \mathbf{q}_e = \frac{j^2}{\sigma} - 3 \mu n \nu_{ei} (T_e - T_i), \quad (6)$$

where the symbols have the following meaning: n is the plasma density, T_e the electron temperature measured in energy units (therefore Boltzmann's constant is omitted), \mathbf{v}_e the collective electron velocity, \mathbf{q}_e the electronic heat flux, j the electric current density, σ the electric conductivity, μ the ratio of the electron to the ion mass, ν_{ei} the electron-ion collision frequency and T_i the ion temperature. The electron viscosity has been neglected here.

In the case of the Q-PIG, this equation may be simplified considerably. First of all, we shall neglect the components of \mathbf{v}_e , ∇T_e and \mathbf{q}_e parallel to the magnetic field. This may be justified by the following simple arguments. In the Q-PIG there are potential walls at the ends of the plasma column (the cathode fall regions) which prevent the electrons to escape axially. Therefore, as far as the cold electrons are considered, v_{ez} may be assumed to be zero. Furthermore, experiments^{11, 12} have shown that the rotating temperature perturbation extends axially closely up to the cathode without changing its form. This means that no parallel temperature gradients should exist. Finally it can be shown¹² that \mathbf{q}_e is approximately proportional to ∇T_e and thus the parallel component, $\mathbf{q}_{e\parallel}$, vanishes together with $\nabla_{\parallel} T_e$.

Returning to Eq. (6) we may replace the complicated ohmic heating term j^2/σ in the case of the Q-PIG by the temperature independent heating term H . Then we have the energy equation in the following form:

$$\frac{3}{2} n \frac{\partial T_e}{\partial t} + \frac{3}{2} n \mathbf{v}_e \cdot \nabla T_e + n T_e \nabla \cdot \mathbf{v}_e + \nabla \cdot \mathbf{q}_e = H - 3 \mu n \nu_{ei} (T_e - T_i), \quad (7)$$

where we have omitted the indices denoting the perpendicular components.

Now we have to specify \mathbf{v}_e and \mathbf{q}_e . The electron velocity may be obtained by solving the equation of motion. In the Q-PIG, this equation is linear in \mathbf{v}_e , since the inertial term may well be neglected¹². The solution of the equation of motion results in the following expression:

$$\mathbf{v}_e = \frac{\nabla p_e \times \mathbf{b}}{n m_e \omega_e} + \frac{\mathbf{E} \times \mathbf{b}}{B} + 0 \left(\frac{\nu_{ei}}{\omega_e} \right), \quad (8a)$$

where p_e is the electronic pressure (nT_e), ω_e the electron cyclotron frequency and \mathbf{b} a unit vector along the magnetic field. The last term contains contributions proportional to powers of the ratio ν_{ei}/ω_e . Since this ratio is of the order of 10^{-2} , the expression (8a) may well be approximated by the first two terms of the right hand side:

$$\mathbf{v}_e \approx \frac{\nabla p_e \times \mathbf{b}}{n m_e \omega_e} + \frac{\mathbf{E} \times \mathbf{b}}{B}. \quad (8b)$$

In order to find \mathbf{q}_e , we will also use an approximate expression for small ratios of ν_{ei}/ω_e :

$$\mathbf{q}_e = -4.66 \frac{\nu_{ei}}{\omega_e} \cdot \frac{n T_e}{m_e \omega_e} \nabla T_e - \frac{5}{2} \frac{n T_e}{m_e \omega_e} \cdot \mathbf{b} \times \nabla T_e; \quad (9)$$

a third term appearing in Braginskii's formula is omitted here since it complicates the calculations to a great extent without modifying but slightly the numerical factor of the first term of Eq. (9) (for a detailed discussion see¹²).

If we now introduce the quantities expressed by (8b) and (9) into (7), we find that the terms due to the diamagnetic drift velocity [the first term of the r.h.s. of Eq. (8b)] are just cancelled out by the term due to the "collision free" heat flux [the last term of Eq. (9)]. This means physically that no energy may flow into or out of a certain volume element of the electron gas unless collisions are involved. This is true for any kind of inhomogeneity in temperature and density regardless whether the gradients are parallel to each other or not. We may therefore, without loss of generality, replace \mathbf{v}_e and \mathbf{q}_e by the following much simpler expressions:

$$\mathbf{v}_e^* = \frac{\mathbf{E} \times \mathbf{b}}{B}, \quad (10)$$

$$\mathbf{q}_e^* = -4.66 \frac{\nu_{ei}}{\omega_e} \cdot \frac{n T_e}{m_e \omega_e} \cdot \nabla T_e \quad (11)$$

and the energy equation then assumes the form:

$$\frac{3}{2} n \frac{\partial T_e}{\partial t} + \frac{3}{2} n \mathbf{v}_e^* \nabla T_e + \nabla \cdot \mathbf{q}_e^* = H - 3 \mu n \nu_{ei} (T_e - T_i). \quad (12)$$

The last step is to account for the electron-neutral collisions by replacing ν_{ei} by the sum $\nu_{ei} + \nu_{en}$ wherever this quantity will appear. In order to see the importance of ν_{en} , consider for instance the last term of Eq. (12). Assuming the temperatures of ions and neutrals to be equal, we have [Eq. (5)]:

$$C(T_e) = 3 \mu n (\nu_{ei} + \nu_{en}) (T_e - T_i).$$

If we calculate the quantities ν_{ei} and ν_{en} for $T_e \approx 2000^\circ \text{K}$, $n_e \approx 10^{12} \text{cm}^{-3}$ and a neutral gas pressure $p \approx 30 \text{mTorr}$, we find that ν_{ei} is of the order of 10^8sec^{-1} whereas ν_{en} is only about 10^7sec^{-1} .

Therefore, as far as we consider only the equilibrium, we may neglect ν_{en} compared with ν_{ei} . However, if we linearize $C(T_e)$, remembering that $\nu_{ei} \sim T_e^{-3/2}$ and $\nu_{en} \sim T_e^{1/2}$, we get:

$$C_1(T_e) \approx -\frac{3}{2} \mu n (\nu_{ei0} - 3 \nu_{en0}) T_1 \quad (13)$$

where the index zero denotes the unperturbed quantities at equilibrium and T_1 is a small perturbation of the equilibrium temperature T_0 . (We have neglected for the moment the ion temperature which is always small compared with T_e .) Now we see immediately that in the linearized term ν_{en0} may by no means be disregarded and furthermore that $C_1(T_e)$ may change its sign depending on the relative magnitudes of ν_{ei0} and ν_{en0} . This will lead to the unstable behaviour of the system considered as will be seen in the next section.

5. Linearization of the Energy Equation

In order to simplify the calculations, let us assume that

1. there are no gradients in z-direction, i. e. $\partial/\partial z \equiv 0$,
 2. density and electric field are rotational symmetric and of zeroth order,
 3. zero order temperature gradients do not exist.
- Furthermore we assume the perturbation of the electron temperature to be of the form:

$$T_1 = T_1(r) \cdot e^{i(\omega t + m\varphi)} \quad (14)$$

where m is the azimuthal mode number and ω is a complex frequency. Eq. (14) means that the radial distribution of the temperature perturbation is stationary whereas it is allowed to propagate azimuthally. We shall further restrict ourselves to a region around the maximum of the temperature distribution, where $\partial T_1/\partial r \approx 0$, i. e. we consider the question whether or not the electron temperature is stable at this particular point. Then it may easily be shown that $\nabla \cdot \mathbf{q}_e$ is reduced (in first order) to an expression proportional to $\partial^2 T_1/\partial r^2 + \partial^2 T_1/r^2 \partial \varphi^2$. Following HOH⁴, we estimate $\partial^2 T_1/\partial r^2$ to be of the order $-k_1^2 T_1$, where k_1 is the inverse characteristic length of the radial distribution of the temperature perturbation. Denoting the radius where the

temperature maximum occurs by r_0 , we have therefore [Eq. (11)]:

$$\nabla \mathbf{q}_{e1} \approx 4.66 \frac{\nu_{ei} + \nu_{en}}{\omega_e} \cdot \frac{n T_0}{m_e \omega_e} \cdot \left(k_1^2 + \frac{m^2}{r_0^2} \right) T_1. \quad (15)$$

This quantity simply accounts for the rate of energy flowing out of a region of size k_1^{-1} and r_0/m , wherein the temperature has been increased by an amount T_1 .

The second term of Eq. (12) which has to be linearized is already given by Eq. (13) except for the contribution due to the finite ion temperature. Including it, we may rewrite Eq. (14) to give:

$$C_1(T_e) = \frac{3}{2} \mu n \nu_{en0} T_1 \left\{ 3 - \frac{T_i}{T_0} - \frac{\nu_{ei0}}{\nu_{en0}} \left(1 - 3 \frac{T_i}{T_0} \right) \right\}. \quad (16)$$

Inserting these two first order quantities, $\nabla \mathbf{q}_{e1}$ and $C_1(T_e)$, into Eq. (12) and using the ansatz (14) we may write the first order energy equation in the following form:

$$\begin{aligned} & \frac{3}{2} n i \omega T_1 + \frac{3}{2} n \frac{E_r}{B} i \frac{m}{r_0} T_1 \\ &= \frac{3}{2} \mu n \nu_{en0} T_1 \left\{ \frac{\nu_{ei0}}{\nu_{en0}} \left(1 - 3 \frac{T_i}{T_0} \right) - 3 + \frac{T_i}{T_0} \right\} \\ & \quad - 4.66 n \nu_{en0} \left(\frac{\nu_{ei0}}{\nu_{en0}} + 1 \right) \frac{T_0}{m_e \omega_e^2} \left(k_1^2 + \frac{m^2}{r_0^2} \right) T_1. \end{aligned} \quad (17)$$

We now see that the first term at the right hand side may act as a heating term provided T_i/T_0 is less than $1/3$ and the ratio ν_{ei0}/ν_{en0} is not too small. Both conditions are fulfilled in the Q-PIG for such parameter values for which the discharge is found to be unstable. Separating this equation into its real and imaginary part, we easily find the following expressions (using the definition $\omega = \omega_R + i \omega_I$):

$$\omega_R = - \frac{E_r}{B} \cdot \frac{m}{r_0}, \quad (18)$$

$$\begin{aligned} -\omega_I &= \mu \nu_{en0} \left\{ \frac{\nu_{ei0}}{\nu_{en0}} \left(1 - 3 \frac{T_i}{T_0} \right) - 3 + \frac{T_i}{T_0} \right\} \\ & \quad - 3.1 \nu_{en0} \left(1 + \frac{\nu_{ei0}}{\nu_{en0}} \right) \frac{T_0}{m_e \omega_e^2} \left(k_1^2 + \frac{m^2}{r_0^2} \right). \end{aligned} \quad (19)$$

Equation (18) means that a temperature distribution of azimuthal mode number m moves with a velocity equal to the $\mathbf{E} \times \mathbf{B}$ drift velocity. This leads, in the laboratory frame of reference, to an oscillation of the electron temperature just given by Eq. (18). The imaginary part as given in Eq. (19) yields the growth rate of the instability and hence by setting ω_I equal to zero, we may find the boundary between stability and instability in a certain parameter space. This is, what has been done ex-

perimentally^{11,12} by changing gas pressure and magnetic field.

We are now able to compare the experimental results to the simple theory presented here. In order to evaluate Eq. (18), we assume that the equilibrium temperature, T_0 , does not depend appreciably on the gas pressure or the magnetic field. These assumptions are suggested from the experimental observations but seem reasonable as well for the following reasons. Keeping the plasma density at a constant value, the heating rate is independent of pressure and the loss rate is proportional mainly to ν_{ei} which also does not depend on the gas pressure. On the other hand, the magnetic field enters in the energy balance only via the heat flux perpendicular to it. However, since in equilibrium no transverse temperature gradients have been observed, a zero order transverse heat flux may not exist and hence the magnetic field may hardly influence the equilibrium temperature.

Let us now define the following abbreviations:

$$\begin{aligned} \nu_{ei0}/\nu_{en0} &\equiv \Gamma; & 3.1 T_0/m_e \omega_e^2 &\equiv \varrho^2; \\ T_i/T_0 &\equiv \vartheta; & k_1^2 + (m^2/r_0^2) &\equiv K_m^2. \end{aligned}$$

Then the condition for ω_I to be zero may be written:

$$\Gamma = \frac{\mu(3-\vartheta) + \varrho^2 K_m^2}{\mu(1-3\vartheta) - \varrho^2 K_m^2}. \quad (20)$$

We may further define a magnetic field B_0 such that the denominator of Eq. (20) becomes zero. The quantity ϱ^2 may then be expressed as

$$\varrho^2 = \varrho_0^2 (B_0/B)^2,$$

where

$$\varrho_0^2 = (3.1 T_0/m_e) \cdot (m_e^2/e^2 B_0^2).$$

Finally we introduce the dimensionless quantity $b = B/B_0$ and find

$$\Gamma(b) = \frac{\mu(3-\vartheta) b^2 + \varrho_0^2 K_m^2}{\mu(1-3\vartheta) b^2 - \varrho_0^2 K_m^2}. \quad (21)$$

This expression means that when B and hence b is varied the ratio of the collision frequencies, $\Gamma(b)$, has to be varied as well in order to keep the growth rate ω_I equal to zero. Since the zero order electron temperature remains constant and the discharge current is adjusted to give constant density, this is achieved in the experiments by changing the gas pressure.

It remains to compare the theoretical result (21) with the experimental one quantitatively. We only have to calculate Γ as a function of b for typical

values of equilibrium temperature T_0 and density. The value of ϑ also needed depends on the ion temperature; since it could not be measured, the ion temperature was estimated using the energy balance for the ion gas¹². For $T_0 = 2000^\circ\text{K}$ one finds $T_i \approx 400^\circ\text{K}$ and hence $\vartheta \approx 1/5$. If we assume $m = 1$, the value of K_m^2 is found to be roughly 1.4 cm^{-2} ; using this value, we calculate B_0 to be 280 Gauss. (In the more complete treatment of Ref.¹² this value is found to be 20% smaller.) Fig. 4 shows the quantity Γ as function of the dimensionless variable b

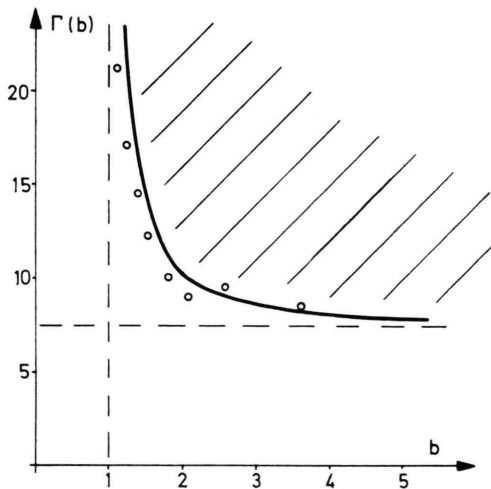


Fig. 4. Stability boundary in a Γ , b plane (solid line) and experimental transition points; the shaded region indicates instability.

(solid curve) as well as the experimental points where the transition between stable and unstable state occurred. The surprising agreement supports greatly the theoretical model treated in this paper.

Finally we should return for a moment to the real part of ω as given by Eq. (18). If we assume $m = 1$, as observed in most cases, the temperature should oscillate at τ_0 with a frequency

$$f = \frac{|\omega_R|}{2\pi} = \frac{1}{2\pi\tau_0} \cdot \frac{E_r(r_0)}{B}. \quad (22)$$

$E_r(r_0)$ is measured to be of order of 0.1 V/cm, B is about 400 Gauss and the radius r_0 , around which the temperature maximum is found experimentally, is 2 cm. Hence we obtain for f about 2 kHz, a value not much higher than observed experimentally. This frequency should decrease with increasing B which in fact has been observed^{11, 12}.

Unfortunately, the dependence on E_r may not easily be checked directly, but supposing that E_r varies similarly as the radial current density j_r , the frequency should increase with the discharge current which is also found experimentally. On the other hand the gas pressure does not greatly influence the observed frequency of instability. This may be explained by the fact that the discharge equilibrium is mainly determined by electron-ion collisions, which do not depend on the neutral gas pressure. Therefore we find that the frequency of the instability as calculated from Eq. (18) is also in reasonable agreement with the experiments.

6. Discussion of the Results

In the preceding section we have seen that the rotational frequency of a temperature perturbation should be caused by the drift velocity of the electrons due to the crossed electric and magnetic fields. In other words, the temperature perturbation will travel just with the gyrocenters of the electrons and it will do so whether or not there is a transverse density gradient. However, this is true only if the electron cyclotron frequency is much greater than the collision frequency of the electrons. Otherwise the perturbation will propagate with a velocity depending on v_{ei}/ω_e and v_{en}/ω_e .

Let us have finally a look back to the zero order energy balance shown graphically in Fig. 2. The function $C(T_e)$ plotted there (solid curve) is quite similar to what is shown in an early paper of GUREVITCH²¹ and a recent paper of VOGEL²². Both authors consider a plasma, the electrons of which are heated by a high frequency electric field. There are to be distinguished two cases according to whether the frequency of the field is higher or lower than the relevant electron collision frequency. In the latter case the heating rate due to the time varying field becomes inversely proportional to the collision frequency. If the electrons collide mainly with ions, this heating rate will be proportional to $T_e^{3/2}$, i. e. the energy input will increase with rising temperature. Equating this energy input to the energy loss by elastic collisions yields an even more catastrophic behaviour of the system with respect to temperature variations. This is shown by Gurevitch who found that there is an abrupt transition between a

²¹ A. V. GUREVITCH, Sov. Phys.-JETP **8**, 271 [1959].

²² R. W. VOGEL, Plasma Phys. **11**, 163 [1969].

state of low electron temperature to one of high temperature, if a certain electric field strength and hence a certain temperature is reached. On the other hand, starting from a high electron temperature and decreasing the electric field strength, the temperature breaks down suddenly and this occurs at a much lower temperature than before; this is what is usually called a hysteresis. Between the transition points there should be a temperature region where the system may not exist, i. e. temperatures in this range are "forbidden".

In the Q-PIG, temperatures in just this region are found to be stable. However, there are some points in which the Q-PIG plasma differs from what was considered by Gurevitch. First, the heating mechanism is independent of the electron temperature and hence the temperature dependence of the energy balance is less pronounced. Second, in the Q-PIG the boundary conditions of the finite plasma certainly play an important role. In particular, the influence of the magnetic field on the energy flux due to transverse temperature gradients is an important factor in the Q-PIG plasma as shown above. Finally, the effects of the cathode sheath, not yet accounted for in the present treatment, should be considered in more detail. This would perhaps clarify the peculiar fact that in the Q-PIG the instability reaches a highly stationary state as indicated by the coherence and reproducibility of the signals.

7. Summary

In a Penning discharge with modified electrode geometry referred to as Q-PIG, low frequency oscil-

lations of the electron temperature have been observed. This phenomenon is investigated theoretically in the present paper. The problem consists of two parts, first to find an appropriate form of the electron energy balance for the equilibrium state and second to investigate how a perturbation of temperature develops in space and time. It is shown that the energy balance of the so called "cold" electrons in the Q-PIG is governed mainly by a heating mechanism involving a group of "hot" electrons and an energy loss process due to elastic collisions of the cold electrons with ions and neutrals. In order to include the effects of spatial temperature distributions as observed in the Q-PIG, the divergence of a transverse heat flux is also taken into account. The energy balance established in this way is then linearized and evaluated by means of a dimensional analysis. It is found that a spatially limited perturbation of the electron temperature may grow exponentially under certain circumstances. It is further shown that this perturbation should be fixed in the electron gas moving with just the $\mathbf{E} \times \mathbf{B}$ drift velocity of the electrons. These results are compared with the experimental ones and remarkable agreement is obtained.

Acknowledgement

The author is indebted to Prof. R. WIENECKE for his active support and many valuable discussions. He is particularly grateful to Dr. H. M. MAYER for his unfailing readiness to discuss the numerous theoretical questions involved.

This work was performed as part of the agreement between the Institut für Plasmaphysik and EURATOM.